

Dynamics of Probe Brane in the Background of Intersecting Fivebranes

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ABSTRACT: This paper is devoted to the study of the dynamics of the Dp-branes, F-strings and M-branes in the background of the system of two stacks of fivebranes in type IIA, IIB and M theory that intersect on the line.

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1. Introduction and Summary

The subject of intersecting branes in string theory is very reach and it has been studied for a long time ². Recently a new interesting approach to this research has been presented in the work [1] where the system consisting of two stacks of the fivebranes in type IIB theory that intersect on $R^{1,1}$ was studied from different points of view. It was found, for example, that

- From the holography arguments one could expect that the dynamics at the $1+1$ intersection of the two sets of fivebranes should be holographically related to a $2+1$ dimensional bulk theory, where the extra dimension being the radial direction away from the intersection. On the other hand the bulk description contains two radial directions away from each set of fivebranes and hence the bulk theory is $3+1$ dimensional. Consequently the corresponding boundary theory is $2+1$ dimensional.
- Very interesting observation that was given in [1] is near horizon symmetry enhancement ³. Naively, looking on the theory at the intersection of the fivebranes one can deduce that this theory should be invariant under $1+1$ dimensional Poincare symmetry $ISO(1,1)$. However, the near horizon geometry describes a $2+1$ dimensional theory with $ISO(2,1)$.

²For review, see [5, 6, 7].

³This observation was also given from different point of view in recent interesting paper [2].

Unfortunately there is not enough place to review all results that were derived in [1] and we recommend this paper for further reading.

In our previous papers [3, 4] we have studied these interesting properties of the I-brane geometry ⁴ from the point of D1-brane probe. We have shown that the enhancement of the near horizon geometry has clear impact on the worldvolume theory of the D1-brane probe. In particular, we have shown that the dynamics of the D1-brane in the near horizon geometry of I-brane can be interpreted as the motion of the probe D1-brane in the near horizon geometry that is sourced by the object that is extended in two spatial dimensions. This result clearly demonstrates an enhancement of the symmetry studied from the I-brane worldvolume theory in [1].

Our study of D1-brane probe in the near horizon geometry of I-brane was based on two different approaches. In the first one we have shown that the D1-brane theory possesses additional scaling like symmetry and we have determined the explicit form of its generator. Using this conserved charge and using also an existence of the conserved energy we were able to solve the equation of motion of the probe D1-brane explicitly. The second way how to study the dynamics of probe D1-brane was based on the transformations given in [1]. Performing these transformations on the worldvolume of D1-brane we were able to map D1-brane action to the form where the enhancement of the symmetry was manifest. We have also shown that these two approaches gave the same picture of D1-brane dynamics in near horizon region of I-brane background.

In this paper we will continue the study of the dynamics of the probe brane in the background consisting of two stacks of fivebranes intersecting on the line. For simplicity we restrict ourselves to the case of time dependent modes on the worldvolume of probe only. We will study the properties of these probes in the near horizon region of various geometries using the approach similar to the one that was given in [1]. Namely, we will try to find transformation that maps the probe action in given near horizon background to the action where an enhancement of the symmetry is manifest. Clearly we can in principle proceed in the same way as in [3, 4] and define the second conserved charge corresponding to the new symmetry that has the form of scaling like symmetry in the original action. Since these two approaches are equivalent we restrict ourselves to the approach based on [1] for its elegance and simplicity.

The plan of this paper is as follows. In the next section we will discuss a D1-brane probe in the background of two stack of D5-branes intersecting on line in type IIB theory. We will see that in the near horizon region the enhancement symmetry is again manifest. We will also see that the dynamics of probe D1-brane with zero

⁴In what follows we use the name "I-brane" for the configuration of two stacks of fivebranes intersecting on a line.

electric flux is trivial as a consequence of the fact that the configuration of D5-branes and D1-brane is supersymmetric.

In section (3) we will study the probe F-string in the background of NS5-branes intersecting on a line in type IIA theory. We will see that again the dynamics of F-string in given background is very simple as a consequence of the fact that the configuration of F-string that is extended along directions parallel with the worldvolumes of NS5-branes is supersymmetric. Then we will address the question of Dp-brane as a probe in I-brane configuration in type IIA theory. Since there is not any stable D1-brane, all worldvolume theories of Dp-branes with ($p \neq 0$) will explicitly depend on the worldvolume spatial coordinate as a consequence of the fact that the Poincare symmetry is preserved in the $R^{1,1}$ subspace only. This observation implies that it is not possible to find configurations that depend on time only. We will also discuss the question of D0-brane as a probe. We find that its dynamics is completely equivalent to the dynamics of D1-brane in the background of I-brane of type IIB theory. We will argue that this is a consequence of the T-duality that relates type IIA to type IIB theory vice versa. Finally in section (4) we will consider a M2-brane probe in the background of two stacks of M5-branes intersecting on line that are however delocalised in the common transverse dimension. We will again see that its dynamics is equivalent to the dynamics of probe F-string in I-brane background in type IIA theory.

Let us now outline our results. We mean that they clearly demonstrate that the approach of the study of the intersecting geometries based on D-brane probe in the near horizon region is very powerful. In fact, the importance of the Dp-brane as a probe has been known for long time [11, 12] ⁵ however as far as we know the study of the I-brane geometry using Dp-brane probe has not been performed so far. For that reason we hope that our modest contribution could be useful for the further study of the properties of I-branes.

2. D1-brane in the D5-D5' background

We start our discussion with the analysis of the dynamics of probe D1-brane in the background of two stacks of D5-brane that intersect on the line. More precisely, we have k_1 D5-branes extended in $(0, 1, 2, 3, 4, 5)$ direction and the set of k_2 D5-branes extended in $(0, 1, 6, 7, 8, 9)$ directions. Let us define

$$\begin{aligned} \mathbf{y} &= (x^2, x^3, x^4, x^5) , \\ \mathbf{z} &= (x^6, x^7, x^8, x^9) . \end{aligned} \tag{2.1}$$

⁵For review, see for example [10].

We have k_1 D5-branes localized at the points \mathbf{z}_n $n = 1, \dots, k_1$ and k_2 D5-branes localized at the points \mathbf{y}_a , $a = 1 \dots, k_2$. Every pairs of fivebranes from different sets intersect at different point $(\mathbf{y}_a, \mathbf{z}_n)$. The background geometry corresponding to this configuration has the form

$$d^2s = (H_1 H_2)^{-1/2} (-dt^2 + (dx_1)^2) + H_1^{-1/2} H_2^{1/2} \delta_{\alpha\beta} dx^\alpha dx^\beta + H_1^{1/2} H_2^{-1/2} \delta_{pq} dx^p dx^q \quad (2.2)$$

together with nontrivial dilaton

$$e^{2\Phi} = (g_1 g_2)^2 H_1^{-1} H_2^{-1} , \quad (2.3)$$

where $\Phi = \Phi_1(\mathbf{z}) + \Phi_2(\mathbf{y})$ and

$$e^{2(\Phi_1 - \Phi_1(\infty))} = \frac{1}{g_1^2} e^{2\Phi_1} = H_1^{-1} , e^{2(\Phi_2 - \Phi_2(\infty))} = \frac{1}{g_2^2} e^{2\Phi_2} = H_2^{-1} . \quad (2.4)$$

Finally, we also have RR three form

$$\begin{aligned} H_{\alpha\beta\gamma} &= -\epsilon_{\alpha\beta\gamma\delta} \partial^\delta \Phi_2 , \alpha, \beta, \gamma, \delta = 2, 3, 4, 5 , \\ H_{pqr} &= -\epsilon_{pqrs} \partial^s \Phi_1 , p, q, r, s = 6, 7, 8, 9 , \end{aligned} \quad (2.5)$$

where

$$\begin{aligned} H_1 &= 1 + \sum_{n=1}^{k_1} \frac{l_s^2}{|\mathbf{z} - \mathbf{z}_n|^2} , \\ H_2 &= 1 + \sum_{a=1}^{k_2} \frac{l_s^2}{|\mathbf{y} - \mathbf{y}_a|^2} . \end{aligned} \quad (2.6)$$

Our goal is to study properties of this background from the point of view of D1-brane probe when $\mathbf{z}_n = \mathbf{y}_a = 0$. In this case (2.6) takes the form

$$H_1 = 1 + \frac{\lambda_1}{|\mathbf{z}|^2} , H_2 = 1 + \frac{\lambda_2}{|\mathbf{y}|^2} , \lambda_1 = N_1 l_s^2 , \lambda_2 = N_2 l_s^2 . \quad (2.7)$$

Let us now consider the probe D1-brane. Recall that the dynamics of the D1-brane is governed by Dirac-Born-Infeld (DBI) action

$$S = -\tau_1 \int d^2\xi e^{-\Phi} \sqrt{-\det \mathbf{A}} , \quad (2.8)$$

where τ_1 is D1-brane tension, ξ^μ , $\mu = 0, 1$ are worldvolume coordinates and where the matrix $\mathbf{A}_{\mu\nu}$ is equal to

$$\mathbf{A}_{\mu\nu} = g_{MN} \partial_\mu X^M \partial_\nu X^N + \partial_\mu A_\nu - \partial_\nu A_\mu , \quad (2.9)$$

where A_μ is worldvolume gauge fields ⁶ and X^M , $M = 0, \dots, 9$ parametrise the position of D1-brane in target spacetime.

Let us now presume that D1-brane is stretched in x^0, x^1 directions. Then it is natural to choose the static gauge where $x^0 = X^0$, $x^1 = X^1$ and hence in the background (2.2) the action (2.8) takes the form

$$S = -\tau_1 \int d^2x e^{-\Phi} \sqrt{-\det \mathbf{A}} , \quad (2.10)$$

where

$$e^{-\Phi} = \frac{H_1^{1/2} H_2^{1/2}}{g_1 g_2} , \mathbf{A}_{\mu\nu} = g_{\mu\nu} + g_{pq} \partial_\mu Z^p \partial_\nu Z^q + g_{\alpha\beta} \partial_\mu Y^\alpha \partial_\nu Y^\beta + \partial_\mu A_\nu - \partial_\nu A_\mu . \quad (2.11)$$

Note that there is also WZ term in the form

$$S_{WZ} = \frac{\tau_1}{2} \int d^2x \epsilon^{\mu\nu} \left(C_{pq} \partial_\mu Z^p \partial_\nu Z^q + C_{\alpha\beta} \partial_\mu Y^\alpha \partial_\nu Y^\beta \right) \quad (2.12)$$

that expresses the coupling of the D1-brane to the Ramond-Ramond two form C .

Now we restrict ourselves on the study of the dynamics of the probe D1-brane in the near horizon region of the I-brane background. As we claimed in the introduction, we can proceed in two ways. The first one that was previously introduced in [3] is based on an existence of an additional symmetry that emerges in the near horizon background for pure time dependent motion of the probe D1-brane. Then using corresponding conserved charge and also using conserved energy of the probe we can solve its equations of motion explicitly. The second approach is based on the beautiful observation [1, 2] that in the near horizon region of I-brane an enhancement of the symmetry of the transverse space emerges. Since it seems to us that the second approach gives more physical insight into the I-brane physics we proceed in the similar way here as well. Namely, we will try to find the transformation in the D1-brane DBI-action in the near horizon region of I-brane that maps this action to the equivalent one where the enhancement of the symmetry will be manifest. As the final remark note that for homogeneous modes the WZ term (2.12) does not contribute to the equations of motion.

Now thanks to the manifest rotation symmetry $SO(4)$ in the subspaces spanned by coordinates $\mathbf{z} = (z^6, z^7, z^8, z^9)$ and $\mathbf{y} = (y^2, y^3, y^4, y^5)$ we will reduce the problem to the study of the motion of two dimensional subspaces, namely we will presume that only following worldvolume modes are excited

$$z^6 = R \cos \alpha , z^7 = R \sin \alpha , \quad (2.13)$$

and

$$y^2 = S \cos \beta , y^3 = S \sin \beta . \quad (2.14)$$

⁶We work in units $2\pi\alpha' = 1$.

We also presume that we have nonzero $\partial_0 A_1 \equiv \dot{A}$ while we work in gauge $A_0 = 0$. Then the matrix \mathbf{A} takes the form

$$\mathbf{A} = \begin{pmatrix} \frac{RS}{\sqrt{\lambda_1 \lambda_2}} \left(-1 + \frac{\lambda_2}{S^2} (\dot{S}^2 + S^2 \dot{\beta}^2) + \frac{\lambda_1}{R^2} (\dot{R}^2 + R^2 \dot{\alpha}^2) \right) & \frac{\sqrt{\lambda_1 \lambda_2}}{RS} \dot{A} \\ -\frac{\sqrt{\lambda_1 \lambda_2}}{RS} \dot{A} & \frac{RS}{\sqrt{\lambda_1 \lambda_2}} \end{pmatrix} \quad (2.15)$$

where $x^0 \equiv t$, $\dot{X} = \partial_0(X)$.

Since the worldvolume theory does not depend on x^1 it is convenient to work with the action density $s \equiv \frac{S}{V}$ where V is the volume of the worldvolume spatial section that in case of D1-brane is simply length L of D1-brane. Using this definition we get

$$s \equiv \frac{S}{V} = - \int dt \frac{1}{g_1 g_2} \sqrt{1 - \frac{\lambda_2}{S^2} (\dot{S}^2 + S^2 \dot{\beta}^2) - \frac{\lambda_1}{R^2} (\dot{R}^2 + R^2 \dot{\alpha}^2) - \frac{\lambda_1^2 \lambda_2^2}{R^4 S^4} \dot{A}^2} . \quad (2.16)$$

Following the work of [1] we now introduce two modes

$$R = e^{\frac{2}{k_1} \phi_1}, S = e^{\frac{2}{k_2} \phi_2} \quad (2.17)$$

and perform the linear transformation in the form

$$\frac{1}{k_1} \phi_1 + \frac{1}{k_2} \phi_2 = A\phi, \phi_1 - \phi_2 = Bx^2 . \quad (2.18)$$

Coefficients k_1, k_2, A, B are fixed by requirements that the kinetic terms for ϕ, x_2 take canonical form $\dot{\phi}^2 + \dot{x}_2^2$. This condition can be obeyed with following values of coefficients

$$A = \frac{1}{2} \sqrt{\frac{1}{\lambda}}, \frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}, B = \frac{\lambda_1}{2}, \quad k_2 = \lambda_2, k_1 = \lambda_1 . \quad (2.19)$$

After performing this transformation the action (2.16) takes the form

$$s = - \int dt \frac{\tau_1}{g_1 g_2} \sqrt{1 - \dot{\phi}^2 - \dot{x}_2^2 - \lambda_1 \dot{\alpha}^2 - \lambda_2 \dot{\beta}^2 - (\lambda_1 \lambda_2)^2 e^{-\frac{1}{\sqrt{\lambda}} \phi} \dot{A}^2} . \quad (2.20)$$

From (2.20) we immediately see that the enhancement of the near horizon geometry takes place even in the D5-D5' background. In fact, from the point of view of D1-brane probe the direction x_2 is equivalent to x^0, x^1 directions. In other words the action (2.20) has the form of the action for a D1-brane that moves in the near horizon region of some $2 + 1$ dimensional object.

Since I-brane background is time independent the energy of D1-brane is conserved and hence it is natural to study its dynamics using the Hamiltonian formalism. For this reason we will now be more general and consider the Lagrangian in the form

$$\mathcal{L} = - \sqrt{V - \sum_i (f_i (\partial_0 \Phi^i)^2 + B_i \partial_0 \Phi^i)} \equiv -\Delta , \quad (2.21)$$

where V is a potential for dynamical fields Φ^i and where f_i and B_i are functions of Φ^j . The conjugate momentum P_i to Φ^i takes the form

$$P_i = \frac{2f_i\partial_0\Phi^i + B_i}{2\Delta} \quad (2.22)$$

and hence

$$\partial_0\Phi^i = \frac{1}{2f_i} (2P_i\Delta - B_i) . \quad (2.23)$$

Then using (2.22) and (2.23) we get the Hamiltonian in the form

$$H = \sum_i P_i\partial_0\Phi^i - \mathcal{L} = \frac{2V - \sum_i B_i\partial_0\Phi^i}{2\Delta} = \frac{2V + \sum \frac{B_i^2}{2f_i}}{2\Delta} - \sum_i \frac{B_i}{2f_i} P_i . \quad (2.24)$$

If we express the Hamiltonian using the canonical variables Φ^i, P_i we get

$$H = \sqrt{\left(V + \sum_i \frac{B_i^2}{4f_i}\right) \left(1 + \sum_i \frac{P_i^2}{f_i}\right)} - \sum_i \frac{B_i P_i}{2f_i} . \quad (2.25)$$

In case of the action (2.20) we have

$$V = \frac{\tau_1^2}{(g_1 g_2)^2} , f_\phi = V , f_{x^2} = V , f_\alpha = \lambda_1 V , f_\beta = \lambda_2 V , f_A = V(\lambda_1 \lambda_2)^2 e^{-\frac{1}{\sqrt{\lambda}}\phi} \quad (2.26)$$

and $B_i = 0$ so that we obtain from (2.25) the Hamiltonian in the form

$$H = \sqrt{\frac{\tau_1^2}{(g_1 g_2)^2} + P_\phi^2 + P_{x^2}^2 + \frac{1}{\lambda_1} P_\alpha^2 + \frac{1}{\lambda_2} P_\beta^2 + \frac{e^{\frac{1}{\sqrt{\lambda}}\phi}}{(\lambda_1 \lambda_2)^2} \pi^2} , \quad (2.27)$$

where $\pi = P_A$. Now the canonical equations of motion for P_α, P_β, P_2 and π can be easily derived from (2.27) and we get

$$\dot{P}_{\Phi^i} = -\frac{\delta H}{\delta \Phi^i} = 0 , \dot{\pi} = -\frac{\delta H}{\delta A} = 0 , \quad (2.28)$$

where $\Phi^i = (\alpha, \beta, x^2)$. We see that the problem reduces to the study of dynamics of ϕ that obeys the canonical equation of motion

$$\dot{\phi} = \frac{\delta H}{\delta P_\phi} = \frac{P_\phi}{E} \quad (2.29)$$

using the fact that H is conserved and equal to the energy E . To solve this equation we express P_ϕ from (2.27) and we get

$$P_\phi = \pm \sqrt{E^2 - \frac{\tau_1^2}{(g_1 g_2)^2} - P_{x^2}^2 - \frac{1}{\lambda_1} P_\alpha^2 - \frac{1}{\lambda_2} P_\beta^2 - \frac{e^{\frac{1}{\sqrt{\lambda}}\phi}}{(\lambda_1 \lambda_2)^2} \pi^2} \equiv \sqrt{G^2 - H^2 e^{\frac{1}{\sqrt{\lambda}}\phi}} , \quad (2.30)$$

where

$$G^2 = E^2 - \frac{\tau_1^2}{(g_1 g_2)^2} - P_{x^2}^2 - \frac{1}{\lambda_1} P_\alpha^2 - \frac{1}{\lambda_2} P_\beta^2, H^2 = \frac{\pi^2}{(\lambda_1 \lambda_2)^2}. \quad (2.31)$$

Then the equation (2.29) takes the form

$$\dot{\phi} = \pm \frac{1}{E} \sqrt{G^2 - H^2 e^{\frac{1}{\sqrt{\lambda}} \phi}}. \quad (2.32)$$

First of all we see that for $\pi = 0$ the equation above has simple solution

$$\phi = t \frac{\sqrt{E^2 - \frac{\tau_1^2}{(g_1 g_2)^2} - P_{x^2}^2 - \frac{1}{\lambda_1} P_\alpha^2 - \frac{1}{\lambda_2} P_\beta^2}}{E} + \phi_0 \quad (2.33)$$

that corresponds to the motion of free particle. In particular, for $P_\phi = 0$ we get $\phi = \phi_0$ and we obtain the solution corresponding to D1-brane at the rest at the distance ϕ_0 from the line of intersection of D5-branes. This is satisfactory result since the configuration of two stacks of D5-branes intersecting on line and D1-brane with the worldvolume paralell with the line of intersection is supersymmetric and hence D1-brane at rest can be localised at any distance from the intersection. Of course this is valid for any R, S as follows from the fact that the original Lagrangian for D1-brane in I-brane background does not contain any potential for R, S when D1-brane does not move. In other words, it can be easily shown that the solution of the equation of motion for vanishing \dot{R}, \dot{S} exists for any R, S .

The situation is different for $\pi \neq 0$. Note that π measures the number of fundamental strings that form the bound state with D1-brane. On the other hand it is well known that the probe F-string in given background breaks all supersymmetries. The manifestation of this fact is the presence of the potential in Hamiltonian (2.27). In spite of this fact we can solve the equation (2.29) and we get

$$e^{\frac{1}{\sqrt{\lambda}} \phi} = \frac{G^2}{H^2} \frac{1}{\sinh^2 \frac{G}{4E\sqrt{\lambda}} t}, \quad (2.34)$$

where we have choosen the initial condition that for $t = 0$ D1-brane reaches its turning point $\dot{\phi} = 0$.

The physical picture of the result of (2.34) is standart: D1-brane leaves the worldvolume of I-brane at $t = -\infty$ reaches its turning point at $t = 0$ and goes back to I-brane for $t = \infty$.

In conclusion of this section we would like to stress the main result determined here. Namely we have shown that the background of two orthogonal stacks of D5-branes posses an ehancement of the symmetry in the near horizon region exactly in the same way as in the case of two stacks of interescting NS5-branes.

2.1 Dynamics of D1-brane near the points $R_t^2 = \lambda_1$, $S_t^2 = \lambda_2$

The analysis performed in [1] suggests an existence of two exception points R_t and S_t whose values have been defined in the title of this section. Namely, it was argued in [1] that free fermions that for zero coupling are supported on $R^{1,1}$ are for nonzero gauge coupling displaced from $R^{1,1}$. More precisely, fermions living on intresection can be replaced by holomorphic current algebra

$$SU(k_1)_{k_2} \times SU(k_2)_{k_1} \times U(1) . \quad (2.35)$$

The carefoul analysis of the low energy effective theory on I-brane performed in [1] leads to following picture. At vanishing coupling $g_1 = g_2 = 0$ $k_1 k_2$ fermions are localised on the at point $r = s = 0$ as suggested by their description in terms of open strings stretched between D5-branes. The turning on the coupling constant moves these fermions from the origin. Different currents that are formed from different fermions move in different directions. In particular, the $SU(k_1)_{k_2}$ move to $s \sim \sqrt{\lambda_2}$. For $SU(k_2)_{k_1}$ one moves to $r \sim \sqrt{\lambda_1}$. The $U(1)$ part is supported at both points.

Our question is whether this interesting behaviour of the low energy I-brane worldvolume theory has some impact on the dynamics of the D1-brane probe in I-brane background. For that reason we will study the dynamics of the D1-brane near the points R_t, S_t defined above. Then it is convenient to write

$$R = r + R_t , r \ll R_t , S = s + S_t , s \ll S_t \quad (2.36)$$

and hence

$$\begin{aligned} H_1 &= 1 + \frac{\lambda_1}{(\sqrt{\lambda_1} + r)^2} \approx 2(1 - \frac{r}{\sqrt{\lambda_1}}) \\ H_2 &= 1 + \frac{\lambda_2}{(\sqrt{\lambda_2} + s)^2} \approx 2(1 - \frac{s}{\sqrt{\lambda_2}}) . \end{aligned} \quad (2.37)$$

With this approximation the DBI action for D1-brane near the points R_t, S_t takes the form

$$s = -\frac{\tau_1}{g_1 g_2} \int dt \sqrt{1 - 2\dot{s}^2 - 2\dot{r}^2 - 2\lambda_1 \dot{\alpha}^2 - 2\lambda_2 \dot{\beta}^2 - 4 \left(1 - \frac{r}{\sqrt{\lambda_1}} - \frac{s}{\sqrt{\lambda_2}}\right)} , \quad (2.38)$$

where we have considered the terms at most linear in r, s and we have restricted on homogeneous modes only. We again introduce two modes x, ϕ defined as

$$r = M_x^r x + M_\phi^r \phi , s = M_x^s x + M_\phi^s \phi , \quad (2.39)$$

where M_j^i , $i = r, s$, $j = x, \phi$ are coefficients that will be determined from following requiraments: Firstly, we demand that the kinetic term $2\dot{r}^2 + 2\dot{s}^2$ takes the cannonical

form in new variables ϕ and x . Secondly, we demand that the following expression

$$\frac{r}{\sqrt{\lambda_1}} + \frac{s}{\sqrt{\lambda_2}} = \left(\frac{M_x^r}{\sqrt{\lambda_1}} + \frac{M_x^s}{\sqrt{\lambda_2}} \right) x^2 + \left(\frac{M_\phi^r}{\sqrt{\lambda_1}} + \frac{M_\phi^s}{\sqrt{\lambda_2}} \right) \phi \quad (2.40)$$

depends on ϕ only. These conditions imply

$$\begin{aligned} M_\phi^r &= \frac{\sqrt{\lambda_2}}{\sqrt{2}\sqrt{\lambda_1 + \lambda_2}}, M_\phi^s = \frac{\sqrt{\lambda_1}}{\sqrt{2}\sqrt{\lambda_1 + \lambda_2}} \\ \frac{M_\phi^r}{\sqrt{\lambda_1}} + \frac{M_\phi^s}{\sqrt{\lambda_2}} &= \frac{1}{\sqrt{2\lambda}}, \frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} \\ M_x^s &= \frac{\sqrt{\lambda_2}}{\sqrt{2}\sqrt{\lambda_1 + \lambda_2}}, M_x^r = -\frac{\sqrt{\lambda_1}}{\sqrt{2}\sqrt{\lambda_1 + \lambda_2}}. \end{aligned} \quad (2.41)$$

Then the action (2.38) takes the form

$$S = -\frac{\tau_1}{g_1 g_2} \int dt \sqrt{1 - \dot{x}^2 - \dot{\phi}^2 - 2\lambda_1 \dot{\alpha}^2 - 2\lambda_2 \dot{\beta}^2 - 4 \left(1 - \frac{\phi}{\sqrt{2\lambda}} \right) \dot{A}^2}. \quad (2.42)$$

Now we can easily study the dynamics of the D1-brane near the points R_t, S_t . We again introduce the Hamiltonian following the genear discussion given in previous subsection. Since for (2.42) we have

$$\begin{aligned} V &= \left(\frac{\tau_1}{g_1 g_2} \right)^2, B_i = 0, f_x = V, f_\phi = V, \\ f_\alpha &= 2\lambda_1 V, f_\beta = 2\lambda_2 V, f_A = 4V \left(1 - \frac{\phi}{\sqrt{2\lambda}} \right) \end{aligned} \quad (2.43)$$

it is clear that the Hamiltonian (2.25) is equal to

$$H = \sqrt{\left(\frac{\tau_1}{g_1 g_2} \right)^2 + P_\phi^2 + P_x^2 + \frac{P_\alpha^2}{2\lambda_1} + \frac{P_\beta^2}{2\lambda_2} + \pi^2 \left(1 - \frac{\phi}{\sqrt{2\lambda}} \right)^{-1}}. \quad (2.44)$$

We again see that P_x, P_α, P_β and π are conserved so that the problem reduces to the study of the dynamics of the mode ϕ . Using (2.44) we get that it obeys the equation of motion

$$\dot{\phi} = \frac{\delta H}{\delta P_\phi} = \frac{P_\phi}{\sqrt{(\dots)}} = \frac{P_\phi}{E}, \quad (2.45)$$

where E is conserved energy. Now from (2.44) we express P_ϕ as

$$P_\phi = \pm \sqrt{E^2 - \left(\frac{\tau_1}{g_1 g_2} \right)^2 - P_x^2 - \frac{P_\alpha^2}{2\lambda_1} - \frac{P_\beta^2}{2\lambda_2} - \pi^2 \left(1 - \frac{\phi}{\sqrt{2\lambda}} \right)^{-1}} \quad (2.46)$$

and hence (2.45) implies

$$\frac{d\phi}{\sqrt{G^2 - H^2\phi}} = \pm dt , \quad (2.47)$$

where we have used the fact that $\phi \ll \sqrt{\lambda}$. Note also that G^2 and H^2 given in (2.47) are defined as

$$G^2 = E^2 - P_x^2 - \frac{P_\alpha^2}{2\lambda_1} - \frac{P_\beta^2}{2\lambda_2} - \pi^2 - \left(\frac{\tau_1}{g_1 g_2} \right)^2 ,$$

$$H^2 = \frac{1}{\sqrt{2\lambda}} \pi^2 . \quad (2.48)$$

By trivial integration of (2.47) we get

$$\phi = \frac{G^2}{H^2} - \frac{1}{H^2} \left(\pm \frac{H^2}{2} t + C \right)^2 , \quad (2.49)$$

Where C is an integration constant. If we choose the initial condition that for $t = 0$ ϕ is equal to zero we get $C = G$. Then the final result takes the form

$$\phi = \pm \frac{t}{2} - \frac{H^2}{4G^2} t^2 . \quad (2.50)$$

We see that for $\pi = 0$ we obtain the trajectory of free particle which is again a consequence of the fact that configuration of two stacks of D5-brane and D1-brane as was defined above is supersymmetric.

We also see that the interesting properties of the I-brane worldvolume theory that were review above do not have any impact on the dynamics of D1-brane probe. This is of course a naturally result since the supergravity background corresponds to the vacuum state of the I-brane worldvolume theory where no I-brane worldvolume modes are excited. For that reason it would be certainly nice to try to generalise the approach presented in [2] to the case of I-brane theory as well. Namely, we can presume that the condensation of the worldvolume modes will lead to the deformation of the background geometry. Then it would be interesting to study the question how the motion of D1-brane probe in such a background depends on the condensation of the I-brane worldvolume fields.

However it is still very interesting that D1-brane near the points R_t, S_t sees an enhancement of the symmetry as well. It would be also nice to understand this observation better.

3. F-string as a probe of I-brane Background in type IIA theory

In this section we will study the dynamics of the macroscopic string in the background of two stacks of NS5-branes intersecting on line in type IIA theory. More precisely,

we have configuration of k_1 NS5-branes extended in $(0, 1, 2, 3, 4, 5)$ direction and the set of k_2 NS5-branes extended in $(0, 1, 6, 7, 8, 9)$ directions. We again define

$$\begin{aligned}\mathbf{y} &= (x^2, x^3, x^4, x^5) , \\ \mathbf{z} &= (x^6, x^7, x^8, x^9) .\end{aligned}\tag{3.1}$$

We have k_1 NS5-branes localized at the points \mathbf{z}_n $n = 1, \dots, k_1$ and k_2 NS5-branes localized at the points \mathbf{y}_a , $a = 1 \dots, k_2$. The supergravity background corresponding to this configuration takes the form

$$\begin{aligned}\Phi(\mathbf{z}, \mathbf{y}) &= \Phi_1(\mathbf{z}) + \Phi_2(\mathbf{y}) , \\ g_{\mu\nu} &= \eta_{\mu\nu} , \mu, \nu = 0, 1 , \\ g_{\alpha\beta} &= e^{2(\Phi_2 - \Phi_2(\infty))} \delta_{\alpha\beta} , H_{\alpha\beta\gamma} = -\epsilon_{\alpha\beta\gamma\delta} \partial^\delta \Phi_2 , \alpha, \beta, \gamma, \delta = 2, 3, 4, 5 \\ g_{pq} &= e^{2(\Phi_1 - \Phi_1(\infty))} \delta_{pq} , H_{pqr} = -\epsilon_{pqrs} \partial^s \Phi_1 , p, q, r, s = 6, 7, 8, 9 ,\end{aligned}\tag{3.2}$$

where Φ on the first line means the dilaton and where

$$\begin{aligned}e^{2(\Phi_1 - \Phi_1(\infty))} &= 1 + \sum_{n=1}^{k_1} \frac{l_s^2}{|\mathbf{z} - \mathbf{z}_n|^2} = H_1(\mathbf{z}) , \\ e^{2(\Phi_2 - \Phi_2(\infty))} &= 1 + \sum_{a=1}^{k_2} \frac{l_s^2}{|\mathbf{y} - \mathbf{y}_a|^2} = H_2(\mathbf{y}) .\end{aligned}\tag{3.3}$$

The dynamics of the fundamental macroscopic string in this background is governed by Nambu-Gotto action

$$S = -\tau_s \int d\tau d\sigma \sqrt{-\det \mathbf{A}} + S_{WZ} ,\tag{3.4}$$

where $\sigma \in (-\infty, \infty)$ and where S_{WZ} expresses the coupling of the string to the two form B . Since we will be interested in the dynamics of the string with the worldsheet alligned along the worldvolume of I-brane and with the time dependent worldsheet fields only the Wess-Zumino term does not contribute to the equation of motions. In (3.4) the matrix \mathbf{A} is

$$\mathbf{A}_{\mu\nu} = \partial_\mu X^M \partial_\nu X^N g_{MN} .\tag{3.5}$$

Then the equations of motion that arrise from (3.4) take the form

$$\frac{1}{2} \partial_K g_{MN} \partial_\mu X^M \partial_\nu X^N (\mathbf{A}^{-1})^{\nu\mu} \sqrt{-\det \mathbf{A}} - \partial_\mu (g_{KM} \partial_\nu X^M (\mathbf{A}^{-1})^{\nu\mu} \sqrt{-\det \mathbf{A}}) = 0 .\tag{3.6}$$

Now let us restrict ourselves to the case of homogeneous worldsheet fields and also presume that the string is alligned along the worldvolume of I-brane. Then it is natural to take

$$\tau = x^0, \sigma = x^1. \quad (3.7)$$

For this ansatz the equation of motion (3.6) for X^1 is trivially satisfied since

$$\partial_\tau \left[g_{11} \left(\mathbf{A}^{-1} \right)^{\sigma\tau} \sqrt{-\det \mathbf{A}} \right] = 0 \quad (3.8)$$

using the fact that for the time dependent modes and for (3.7) the matrix \mathbf{A} takes the form

$$\mathbf{A}_{\tau\tau} = g_{\tau\tau} + g_{IJ} \partial_\tau X^I \partial_\tau X^J, \mathbf{A}_{\tau\sigma} = \mathbf{A}_{\sigma\tau} = 0, \mathbf{A}_{\sigma\sigma} = g_{\sigma\sigma}. \quad (3.9)$$

Since $g_{\tau\tau} = -1$, $g_{\sigma\sigma} = 1$ we immediatelly see that the fundamenal string with vanishing velocity can be localised in any position labeled with (\mathbf{z}, \mathbf{y}) since (3.4) does not contain any potential term and all terms as $g_{IJ} \dot{X}^I \dot{X}^J$ vanish for $\dot{X}^I = 0$. This result is in agreement with the well known fact that the configuration of NS5-branes and fundamental strings with the paralell worldvolumes are supersymmetric.

Let us now concetrate on the configuration of coincident NS5-branes where we have

$$H_1 = 1 + \frac{\lambda_1}{|\mathbf{z}|^2}, H_2 = 1 + \frac{\lambda_2}{|\mathbf{y}|^2}, \quad (3.10)$$

and

$$\lambda_1 = k_1 l_s^2, \lambda_2 = k_2 l_s^2. \quad (3.11)$$

As in the previous section we use the manifest rotation symmetries $SO(4)$ and introduce modes

$$z^6 = R \cos \alpha, z^7 = R \sin \alpha, \quad (3.12)$$

and

$$y^2 = S \cos \beta, y^3 = S \sin \beta. \quad (3.13)$$

For this ansatz the string action takes the form

$$s \equiv \frac{S}{V} = -\tau_s \int d\tau \sqrt{1 - H_1(\dot{R}^2 + R^2 \dot{\alpha}^2) - H_2(\dot{S}^2 + S^2 \dot{\beta}^2)}. \quad (3.14)$$

In what follows we will be interested in the study of the dynamics of the fundamental strings in the near horizon region where $\frac{\lambda_1}{R^2} \gg 1$, $\frac{\lambda_2}{S^2} \gg 1$. In this region the action simplifies as

$$s = - \int dt \sqrt{1 - \frac{\lambda_1}{R^2} \dot{R}^2 - \lambda_1 \dot{\alpha}^2 - \frac{\lambda}{S^2} \dot{S}^2 - \lambda_2 \dot{\beta}^2}. \quad (3.15)$$

Now using the transformations

$$R = e^{\frac{\phi_1}{\sqrt{\lambda_1}}}, S = e^{\frac{\phi_2}{\sqrt{\lambda_2}}} \quad (3.16)$$

the action takes simple form

$$s = - \int dt \sqrt{1 - \dot{\phi}_1^2 - \dot{\phi}_2^2 - \lambda_1 \dot{\alpha}^2 - \lambda_2 \dot{\beta}^2} \quad (3.17)$$

that explicitly demonstrates the fact that F-string and NS5-brane are supersymmetric configurations since the dynamics that follows from the action above corresponds to the motion of free particle.

In this section we studied the dynamics of the fundamental string probe in the I-brane background. We have seen that this dynamics is trivial since the probe F-string when it is at the rest, forms supersymmetric configuration with the background I-brane. Even if the dynamics is not very interesting we mean that it was useful to review it here since when we consider I-brane in the type IIA theory then the only stable probe whose worldvolume fits the worldvolume of I-brane is F-string. For D2-brane one of its spatial dimensions has to extend in the direction transverse to the I-brane and hence the D2-brane DBI action explicitly depends on the worldvolume spatial coordinate. Then one can expect that the ansatz with homogeneous fields does not solve the equation of motion that arise from the DBI action for D2-brane in I-brane background.

On the other hand there is a possibility to probe I-brane with D0-brane whose dynamics in I-brane background is governed by the action

$$S = -\frac{\tau_0}{g_1 g_2} \int dt \frac{1}{\sqrt{H_1 H_2}} \sqrt{1 - H_1(\dot{R}^2 + R^2 \dot{\alpha}^2) - H_2(\dot{S}^2 + S^2 \dot{\beta}^2) - \dot{Y}^2}, \quad (3.18)$$

where Y is the mode that describes propagation of D0-brane along the I-brane worldvolume. Now it is easy to see that the dynamics of D0-brane is equivalent to the dynamics of the probe D1-brane with conserved electric flux. This can be easily seen from the fact that $P_Y = \frac{\delta L}{\delta \dot{Y}}$ is conserved and it is related to the conserved electric flux on the worldvolume of D1-brane through T-duality. More precisely, if we compactify the theory on the circle of radius R along the worldvolume of I-brane in type IIB theory we can see that now the configuration of I-brane with the probe of D1-brane is T-dual to the I-brane on the dual circle and D0-brane that moves with constant momentum along this dual circle in type IIA theory. These arguments are based on the fact that T-duality acts along worldvolumes of the background of NS5-branes and it is well known that under this T-duality NS5-brane in type IIA(IIB) theory is mapped to NS5-brane in IIB(IIA) theory ⁷.

Using this duality it is now easy to argue that D0-brane in the near horizon region again “feels” an enhancement of the symmetry of the near horizon geometry [1] in the same way as it was shown in case of D1-brane in near horizon region of I-brane background in [3].

⁷For review of this topic and extensive list of references, see, for example [8].

4. M5-branes overlapping in a string

The solution studied in previous section that consists two stacks of orthogonal NS5-branes in type IIA theory can be naturally uplifted to the solution in M-theory that has the form [9]

$$ds^2 = (H_1 H_2)^{-1/3} (-(dx^0)^2 + (dx^1)^2) + H_1^{-1/3} H_2^{2/3} \delta_{\alpha\beta} dx^\alpha dx^\beta + H_1^{2/3} H_2^{-1/3} \delta_{pq} dx^p dx^q + H_1^{2/3} H_2^{2/3} (dx^{10})^2, \quad (4.1)$$

where H_1, H_2 have the same form as in the previous section. There is also nonzero three form C however we will not need to know its explicit from following reason. We will be interested in the dynamics of the probe M2-brane with the worldvolume stretched along the line of intersection of M5-branes and also along the direction x^{10} . Since we also restrict ourselves to the homogeneous modes it turns out that the coupling of C to the M2-brane vanishes.

The background configuration (4.1) describes two stack of fivebranes in the $(1, 2, 3, 4, 5)$ and in $(1, 6, 7, 8, 9)$ directions that overlap in a string in (1) direction. Note that although the functions H_1, H_2 depend on the relative transverse directions, they are translationally invariant in the overall transverse direction x^{10} .

Now the natural probe of this background is M2-brane whose DBI part of the action takes the form

$$S = -\tau_{M2} \int d^3\xi \sqrt{-\det g_{\mu\nu}}, \quad g_{\mu\nu} = g_{MN} \partial_\mu X^M \partial_\nu X^N, \quad (4.2)$$

where τ_{M2} is M2-brane tension and $\xi^\mu, \mu = 0, 1, 2$ label the worldvolume of M2-brane.

If we fix the gauge as $\xi^0 = X^0 = t, \xi^1 = X^1, \xi^2 = X^{10}$ and then restrict to the homogeneous modes we obtain the action $s \equiv \frac{S}{V_2}$ in the form

$$s = -\tau_{M2} \int dt \sqrt{-g_{11} g_{1010} (g_{00} + g_{IJ} \partial_0 X^I \partial_0 X^J)}, \quad (4.3)$$

where V_2 is the volume of the spatial section of M2-brane worldvolume and $X^I, I = 2, \dots, 9$ label the positions of M2-brane in a space transverse to its worldvolume. Now if we insert the components of the metric given in (4.1) and introduce the modes R, S, α, β in the same way as in the previous sections we obtain the action

$$s = -\tau_{M2} \int dt \sqrt{1 - H_1 (\dot{R}^2 + R^2 \dot{\alpha}^2) - H_2 (\dot{S}^2 + S^2 \dot{\beta}^2)}. \quad (4.4)$$

As we can expect this action has the same form as the action for fundamental F-string in I-brane background studied in the previous section. This follows from the fact that the background (4.1) does not depend on x^{10} . Moreover, M2-brane is extended in x^{10} as well but we have presumed that all worldvolume modes depend on

time only. Put differently, the configuration of two stacks of intersecting M5-branes and a probe M2-brane with one spatial direction parallel with the line of intersection of two stacks of M5-branes and the second one that is extended in x^{10} direction is supersymmetric [9]. Then it follows that the action (4.4) of such a probe in the near horizon region after performing the same transformations as in the case of F-string probe in I-brane background in type IIA theory corresponds to the action for free particle and hence its dynamics is trivial. Since this procedure is completely standard we will not perform it here.

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